Simultaneous Latent Fingerprint Recognition

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Abstract

Simultaneous latent fingerprints are clusters of friction ridge impressions deposited concurrently in a crime scene. The analysis of these impressions is a complex task to infer individualization, exclusion or categorize as inconclusive. The problem is further compounded when distinctive features in each latent fingerprint in the cluster are of varying quality or none of the fingerprint has the requisite number of features to reliably arrive at a conclusion. Recently, SWGFAST (Scientific Working Group on Friction Ridge Analysis, Study and Technology) proposed a draft standard for simultaneous impression examination. The approach is manual and requires known reference ten-print for comparing with an unknown simultaneous latent fingerprints. This paper, proposes a semi-automatic approach to process and analyze simultaneous latent fingerprints. The algorithm demonstrates that comparisons can be made from a database of ten-prints for a more comprehensive search. The algorithm is validated experimentally using a database of simultaneous fingerprints by comparing the time taken to arrive at a decision and the recognition accuracy.

Key words:
Fingerprint recognition, Simultaneous latent impression, Likelihood ratio, Support vector machine

1 Introduction

Recently the forensic community has been motivated to reliably analyze simultaneous latent fingerprints, that can be lifted when two or more friction

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ridge impressions are deposited on an object in a single act of touch (Fig. 1), and render an outcome signifying individualization, exclusion or inconclusive. In 2005, the Supreme Court of Massachusetts rendered a decision in the case of Commonwealth v Patterson [1]. It involved the murder of a Boston police detective. As part of the investigation, four latent prints were collected from the detective's vehicle. An analysis of the four prints revealed that all impressions were deposited simultaneously by the defendant. Furthermore, none of the latent impressions in the cluster had sufficient quality or quantity of features/details to conclude individualization using the ACE-V (referred within the profession as Analysis, Comparison, Evaluation and Verification) methodology. The comparison with the defendant's fingerprints showed that there were only six, five, two, and zero feature points of similarity in each of the latent prints in the cluster. However, having established that the simultaneous impressions belonged to the defendant, the fingerprint examiner chose to aggregate all 13 points of similarity from multiple fingers to conclude individualization. The court found that the State failed to prove the scientific reliability of applying ACE-V methodology to simultaneous fingerprint impressions because it did not satisfy the Daubert analysis [2] for acceptance of evidence especially when none of the latent prints in the cluster could be individually matched. The court also raised several questions regarding the acceptance of the underlying theory and application in the scientific community, the lack of formal testing, the lack of documentation of results in peer reviewed forensic publications, unavailability of data on potential error rates, and the lack of guidelines and standards by the Scientific Working Group on Friction Ridge Analysis, Study and Technology (SWGFAST) [3].

Since the decision on the Commonwealth v Patterson case, progress has been made on two fronts. First, the most recent working draft standard for simultaneous impression examination was developed and released by SWGFAST. The document includes standards for (a) *analyzing two or more friction ridge impressions to determine whether they are consistent with having been deposited on an object simultaneously*, (b) *analyzing a simultaneous impression to determine how it will be compared*, (c) *conclusions from the comparison of a simultaneous impression with known exemplars*, (d) *verification of conclusions*, (e) *documenting the examination*, and (f) *reporting results* [3].

Second, a pilot study was undertaken by Black using 30 latent impressions to study whether latent print examiners could correctly determine if the impressions were simultaneous [4]. The experiment was well designed to include a set of simultaneous impressions collected from individual donors and another set of impressions collected from multiple donors to falsely create the appearance of simultaneous impressions. The images were of good quality and contained large quantities of information. The results compiled from 31 fingerprint examiners considered attributes such as orientation, deposition pressure, distortion, and anatomical spatial tolerances to determine if the impressions were simul-

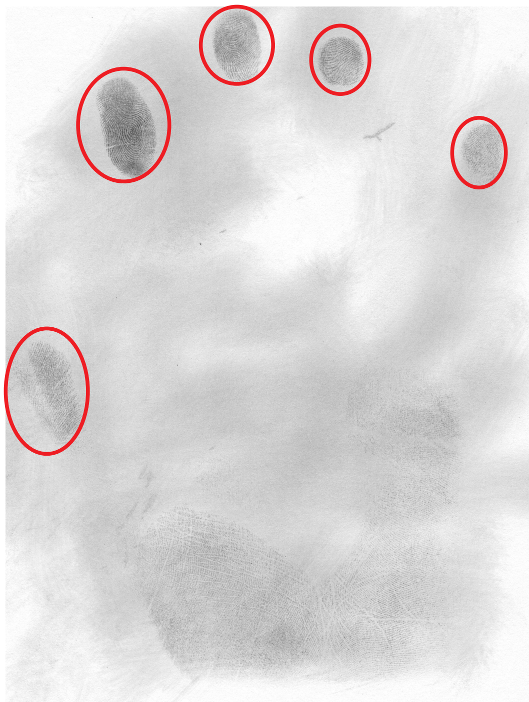

Fig. 1. An example of simultaneous latent fingerprint image.

taneous or not. The results showed that 88% of the time the examiners were able to correctly determine whether two or more latent fingerprint impressions were deposited at the same time.

The SWGFAST standard [3] for examining simultaneous latent fingerprint impressions was the first ever to formally address the significance of this topic and an attempt to establish a systematic process for latent fingerprint examiners to analyze latent simultaneous friction ridge impressions. This process is manual and time consuming. The problem is further compounded when there is no reference latent fingerprint impression available to compare the simultaneous latent fingerprint impressions collected from the crime scene. Also, the pilot study performed by Black [4] is an important step in establishing if the latent print examiners can correctly determine the simultaneity of the impressions. However, because the database was small in size and contained good quality latent prints, the study provides very little information for generating statistical results on the error rates that could be used in the court or by forensic fingerprint examiners.

1.1 Literature Review

Fingerprint recognition can be divided into two tasks: verification and identification. Fingerprint verification is used to verify the identity of an individual by 1:1 matching whereas identification is used to establish the identity by 1:N

matching. Fingerprint identification thus becomes more challenging than verification because of high system penetration and false acceptance rate. In literature, there are three methods to perform identification [5]. The *brute force* method matches probe image to all gallery images. *Classification* matches probe image to gallery images with corresponding class (left loop, right loop, whorl, arch, and tented arch). *Indexing* matches parameters of probe image with gallery images to enable sublinear time lookup.

Brute force identification and classification methods have certain limitations. For example, in applications such as law enforcement and border security where database contains millions of images, the first method would require significantly large number of comparisons and is not feasible. Classification method divides the database into different classes depending on level-1 features or some other classification technique. This method reduces the number of images to a certain extent but since the number of features for classification is small, each class still contains large number of images. In both cases, large number of gallery images lead to high system penetration coefficient and false accept rate.

To address the challenges of these two methods, researchers have proposed indexing based identification algorithms. Database indexing speeds the identification process by reducing the number of required matches without compromising the verification performance. Germain *et al.* proposed a flash algorithm for fingerprint indexing [6]. Bebis *et al.* proposed the Delaunay triangulation of minutia points to perform fingerprint indexing [7]. Boer *et al.* used the registered directional field estimate, FingerCode and minutiae triplet along with their combination to index fingerprint databases [8]. Bhanu and Tan [9] generated minutiae triplets and used angles, handedness, type, direction, and maximum side as the features for indexing. They also applied some constraints on minutiae selection to avoid spurious minutiae. Further, Li *et al.* [10], Feng and Cai [11], and Choi *et al.* [12] proposed indexing algorithms using level-2 fingerprint features. Vatsa *et al.* [13] proposed an indexing algorithm that integrates level-1, level-2 and level-3 features [1] using Delaunay triangulation.

Currently in the literature, to the best of our knowledge, there is no algorithm or automated system that compares simultaneous fingerprint impressions with reference fingerprints stored in a database. The major reasons for the problem not being studied are:

(1) Lack of simultaneous latent fingerprint database that includes rolled fingerprints along with varying quality of simultaneous latent impressions.
(2) Lack of automatic/semi-automatic feature extraction algorithm (friction

[1] Fingerprint features are divided into three levels: level-1 features (example: whorl and arch), level-2 features (example: ridge ending and bifurcation), and level-3 features (example: pores and ridges) [5], [14].

ridge analysis) for latent fingerprint impressions of varying quality and quantity.
(3) Lack of an algorithmic approach to analyze simultaneous impressions.
(4) Lack of statistical study in simultaneous impressions to analyze error rates when individual prints in simultaneous impressions have insufficient quantity and quality of detail and therefore require all features to be combined in aggregate to reach a conclusion.

1.2 Research Contribution

Existing automatic fingerprint recognition algorithms, generally, do not have the capability to authenticate simultaneous latent impressions. In this research, we investigate this important research issue and propose a semi-automatic approach. Specifically, the contributions of this research are listed as follows:

- The objective of this research is to develop a semi-automatic approach that is capable of comparing simultaneous impressions of an unknown individual against a database of known full prints. The proposed approach utilizes fingerprint features, likelihood ratio and support vector machine (SVM) to identify a given simultaneous latent impression.
- A database is prepared that contains 300 simultaneous latent fingerprint impressions. This database will be available to the research community in the future to encourage research in this important area.
- A study on latent simultaneous fingerprint impressions is also performed to determine the error rate when points of similarity from noncontiguous simultaneous impressions are treated in aggregate for comparison and identification purposes.

The next section presents details of the simultaneous latent fingerprint database prepared as a part of this research. Section 3 presents a semi-automatic approach for identifying simultaneous impressions and Section 4 presents the experimental results and analysis.

2 Simultaneous Latent Fingerprint Database

Since there is no publicly available database that contains simultaneous latent impressions, the authors created a database of 300 simultaneous latent impressions captured from different surfaces such as plastic and glass. For simultaneous impressions, input from latent forensic examiners is used to ensure that the database clearly reflects the images collected in real-world scenarios. Examples include the number of fingerprint combinations, specific combination of fingerprints, and different orientation of simultaneous impressions. Latent fingerprints are developed using powder method and the images are scanned using flat-bed scanner at 1000 ppi. We also capture the corresponding rolled ten-prints using optical sensor at 1000 ppi. The database is collected from 150 individuals. From every individual, two instances of simultaneous latent impressions are captured and one instance of rolled ten-print images is captured using an optical scanner. Hence, there are 300 test/probe cases for simultaneous latent fingerprint impressions and 150 rolled ten-prints in the gallery. Further, the gallery is augmented with rolled ten-prints from 100 different individuals; thus the size of gallery database is 250 ten-prints. Fig. 2 shows a sample of simultaneous latent fingerprint impressions lifted from a ceramic plate.

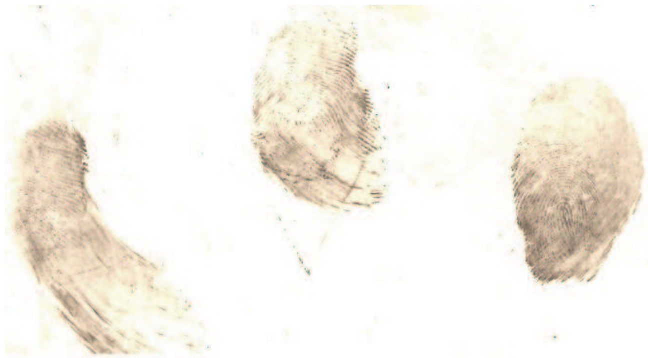

Fig. 2. An example from the database: simultaneous latent fingerprint image with three friction ridge impressions.

3 Proposed Simultaneous Latent Fingerprint Recognition Algorithm

In this research, we developed a semi-automatic approach to process and analyze simultaneous fingerprint impressions. Fig. 2 illustrates the steps involved in the proposed approach and compares it with the existing manual approach.

3.1 Existing Approach

In the existing approach, following the ACE-V methodology, the analysis first determines if the friction ridge impressions are consistent with a simultaneous impression by using factors such as orientation, deposition pressure, and anatomical spatial tolerance of each impression. The next step determines if an impression will stand alone (i.e. has sufficient features to conclude individualization), or if impressions (that do not have sufficient details) must be grouped together and compared in aggregate, or if an impression (that has no

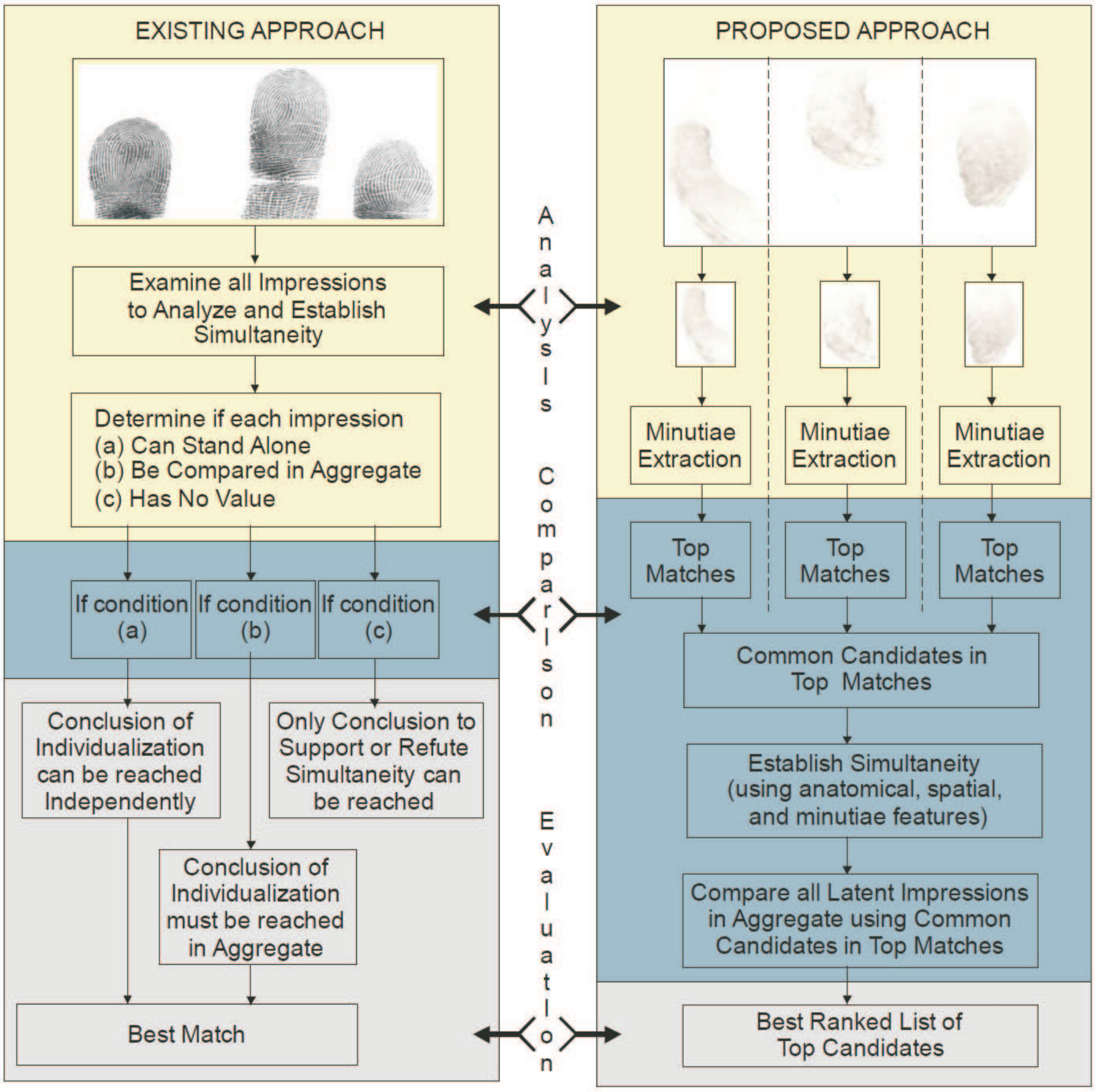

Fig. 3. Proposed approach for Processing Simultaneous Fingerprint Impressions and comparison with existing manual approach.

detail of any value) only supports simultaneity. Each friction ridge impression that is determined to be of value is compared with corresponding known exemplars. A conclusion is reached by evaluating the stand alone impressions. Friction ridge impressions that do not stand alone must be compared in aggregate to reach a conclusion. While this approach is suitable for humans to process these steps, it is challenging to automate the process.

3.2 Proposed Approach

In the preliminary phase of this research, we developed a semi-automatic approach to speed-up the identification process of simultaneous fingerprints. The proposed two stage approach is designed as an add-on tool to augment the capability of existing fingerprint recognition systems.

First Stage: In the first stage, each friction ridge impression in the given cluster is isolated and the fingerprint examiner manually marks the minutiae and level-3 features such as ridge contours, dots and incipient ridges [2]. Irrespective of whether an impression has sufficient details (stand alone), or limited details (compared in aggregate) or has no value (only supports simultaneity), each friction ridge impression is compared with all available exemplars in the database to generate a set of top matches. The gallery-probe minutiae are matched using a dynamic bounding box based matching algorithm [16] and level-3 features are matched using the Mahalanobis distance [13]. Let n be the number of fingerprint images in the simultaneous impression cluster and M_n be the set of top matches for the n^{th} latent impression. From each set of top matches, a subset, M_c, comprising of common candidates is identified, i.e.

$$M_c = M_1 \cap M_2 \cap \cdots \cap M_n \tag{1}$$

Second Stage: In the second stage, these selected candidates are analyzed by the forensic examiner to ascertain simultaneity. This analysis is based on spatial, frequency, and anatomical features to establish if all friction ridge impressions in the cluster are from the same person and deposited at the same time. Here, we would like to emphasize that establishing simultaneity is accomplished by the SWGFAST guidelines [3]. Once simultaneity is established, we compare all friction ridge impressions of the unknown simultaneous impression with each print identified in the subset of common candidates to compute a ranked order of candidates. Here we use likelihood ratio [17] based SVM fusion to attune the top matches.

Let $\mathbf{x} = [x_1, x_2, \cdots, x_n]$ be the match scores corresponding to all n constituent fingerprints in a simultaneous impression cluster and first candidate in the top (gallery) list. The densities of the genuine and imposter scores ($f_{gen}(\mathbf{x})$ and $f_{imp}(\mathbf{x})$, respectively) are estimated. In the proposed approach, it is assumed that the match scores follow a Gaussian distribution, i.e.,

$$f_j(x_i, \overline{\mu}_{ij}, \overline{\sigma}_{ij}) = \frac{1}{\overline{\sigma}_{ij}\sqrt{2\pi}} exp\left[-\frac{1}{2}\left\{\frac{x_i - \overline{\mu}_{ij}}{\overline{\sigma}_{ij}}\right\}^2\right] \tag{2}$$

[2] Note that the gallery images are rolled ten-prints captured from optical sensor; therefore the features can be extracted using the automatic feature extraction process. For the gallery images, level-2 minutiae are extracted using the ridge tracing minutiae extraction algorithm [15] and level-3 features are extracted using the curve evolution based algorithm [13].

where $\overline{\mu}_{ij}$ and $\overline{\sigma}_{ij}$ are the mean and standard deviation of the i^{th} classifier corresponding to the j^{th} element of Θ. While this is a very strong assumption, it does not impact the performance of fusion system in the context of this application. We compute the likelihood ratio $S_i = \frac{f_{gen}(x_i)}{f_{imp}(x_i)}$ pertaining to each constituent impression. The resultant value S_i is used as input to the dual ν-SVM (2ν-SVM) fusion algorithm. *Further, utilizing the 2ν-SVM classifier for fusion addresses the limitations of the likelihood test-statistic if the input data does not conform to the Gaussian assumption (which is usually the case).*

In 2ν-SVM [3] training, likelihood ratios induced from the match scores and their labels are used to train the 2ν-SVM for fusion. Let the labeled training data be represented as $Z_i = (F_i, y)$, where, i represents the i^{th} latent fingerprint in the simultaneous impression. For each match score, the class label $y \in \Theta$ (or $y \in (+1, -1)$; here, $+1$ represents the genuine class and -1 represents the impostor class). n 2ν-SVMs are trained using these labeled training data; one for each latent fingerprint. Further, standard procedure is followed for learning the parameters such as ν parameters and radial basis kernel parameters. The training data is mapped to a higher dimensional feature space such that $Z \to \varphi(Z)$ where $\varphi(\cdot)$ is the mapping function. The optimal hyperplane which separates the data into two different classes in the higher dimensional feature space can be obtained using SVM learning approach (Appendix A).

While testing a query, the fused score of a test pattern $[F_i], (i = 1, 2, \cdots, n)$ is defined as,

$$g(F_{fused}) = \sum_{i=1}^{n} g(F_i), \tag{3}$$

where,

$$g(F_i) = w_i \varphi(F_i) + b_i. \tag{4}$$

Here, w_i and b_i are the parameters of the 2ν-SVM hyperplane. The solution of Equation (3) is the signed distance of F_{fused} from the separating hyperplane [18]. Using the proposed algorithm, fused score computed for the top M_c matches (gallery-probe pairs are generated by pairing the probe image with the top M_c gallery matches). Finally, these M_c values are sorted in the descending order and the new ranking is used for identification.

[3] Details of 2ν-SVM are provided in Appendix - A.

4 Experimental Results and Discussion

The performance of the proposed algorithm is evaluated using the cumulative match characteristic (CMC) curve that is generated by computing the identification accuracy at different top match ranks. For training the proposed approach, NIST special databases (27, 29, and 30) were used to learn the Gaussian Model for density estimation and 2ν-SVM parameters. Note that, using the training database, these parameters are obtained empirically by computing the verification accuracy for different combination of parameters. For example, the radial basis function (RBF) kernel with RBF parameter $\gamma = 4$ yields the best accuracy for the SVM fusion. Further, as described in Section 2, 250 ten-prints are used as the gallery and 300 simultaneous latent prints are used as the probe.

Along with computing the identification performance of the proposed semi-automatic algorithm, we compare the performance with the approach in which latent impressions are processed as stand-alone and with the performance of forensic examiners (complete manual process). We also perform an experiment in which top 10 matches obtained from the proposed semi-automatic approach are shown to the forensic examiners and they provide the closest match. Fig. 4 and Table 1 illustrate these experimental results. The key results and analysis of our experiments are summarized below.

- Simultaneous latent fingerprint impressions usually have less than 12 minutiae in each of the images in the cluster. In our database, maximum number of minutiae in a latent image is 21 and minimum is zero. On the other hand, as shown in Table 2, each simultaneous latent impression (i.e. more than one latent fingerprint) has an average of 46 minutiae. This comparison clearly shows that the simultaneous latent impressions contain more discriminative features than single latent fingerprint.
- Out of 300 probe cases, there are 231 cases in which none of the individual fingerprints in a simultaneous impression are categorized as stand alone. For such cases, the proposed two stage approach improves the performance significantly. Specifically, as shown in Fig. 4, rank-1 identification accuracy improves by around 37%.
- Even though there is a significant improvement, rank-25 accuracy is not more than 80% on a gallery database of 250 ten-prints. Since there is a significant difference between the quality of simultaneous latent impression and gallery rolled ten-print (obtained using an optical scanner), the rank 25 accuracy of around 78% is considerably very high. This experiment also shows that level-3 features (for rolled fingerprint) computed using the automatic feature extraction algorithm [13] and latent fingerprint features manually marked by fingerprint examiners are in accordance, thereby providing a high identification accuracy.

- In the proposed approach, we use both level-2 and level-3 features for matching. Incorporating level-3 features, generally, improves the identification performance for cases in which latent impressions have fewer minutiae but some dots, incipient ridge, and ridge contours are available. Moreover, in our experiments, we observe that among level-3 features, dots are the most stable and pores (specifically pore shape and size) are the least stable features.
- On a gallery size of 250 ten-prints, computational time [4] of the proposed approach for identifying a simultaneous impression is around 07 minutes excluding manual operations (including manual operations, it is around 38 minutes). When compared with the performance of forensic examiners, the time required by the completely manual process is around 3 hours for identifying the impression. However, note that the time taken by forensic examiners depends on several factors such as quality of impression, size of gallery database, and experience of the examiner.
- As mentioned previously, we provided the top 10 matches obtained by the proposed semi-automatic approach to the forensic examiners and they analyzed and identified the closest match among the top 10 matches. We observe that forensic examiners yield best rank-1 accuracy. They also emphasize that the proposed approach reduces the search space, thereby making the complete process relatively fast compared to the manual process (Table 1).
- Currently, the database has only simultaneous impressions without any disjunct impressions and for establishing simultaneity, we rely on manual process. We envision that, in future, with extended database containing both simultaneous and disjunct impressions, we will perform a large scale statistical evaluation to determine individuality, usefulness and applicability of simultaneous latent fingerprint impressions in forensic applications.

5 Conclusion and Future Work

This paper formally introduces an important research problem of "simultaneous latent fingerprint identification". We also present preliminary results of the proposed semi-automatic approach for identifying simultaneous fingerprints. The proposed approach is compatible with existing fingerprint identification systems because each friction ridge impression in the cluster is also processed and analyzed individually. It follows the well established ACE methodology [3] to analyze simultaneous latent fingerprint impressions. The proposed approach effectively processes simultaneous impressions even when none of the friction ridge impressions stand alone so all of them must be compared in aggregate to reach a conclusion. The results show that even with a large database of unknown fingerprints to compare with, the algorithm identifies the top matches

[4] Time is computed on a 2.4 GHz Pentium Duo Core processor with 4 GB RAM under MATLAB environment.

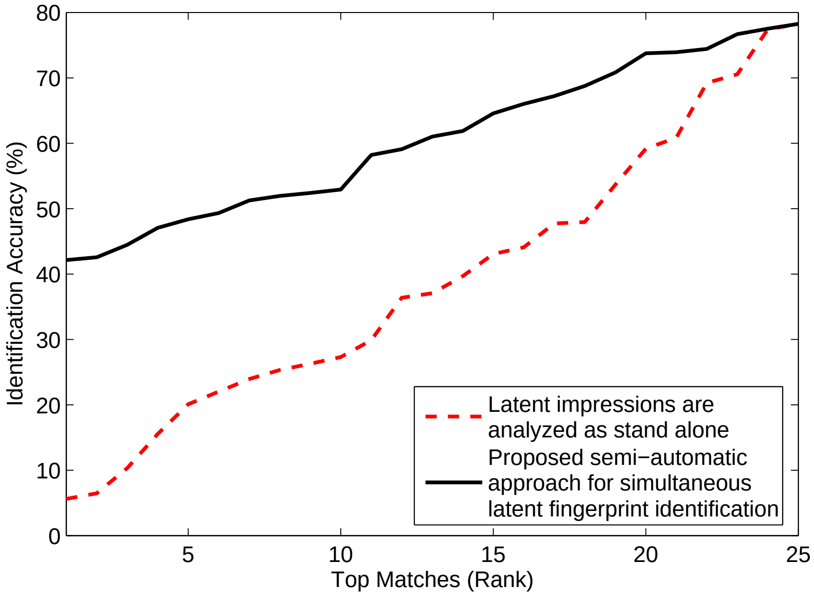

Fig. 4. Cumulative match characteristic curves comparing the performance of the proposed approach for simultaneous latent fingerprint identification with the standard approach when the latent prints are analyzed as stand alone.

Table 1
Identification performance of the proposed approach for simultaneous latent fingerprint identification.

<table>
<tr><th>Experiment</th><th>Algorithm</th><th>Rank 1 Accuracy (%)</th><th>Average Time (Minutes)</th></tr>
<tr><td rowspan="4">Rolled Ten-print with latent simultaneous impression</td><td>Forensic examiners (manual)</td><td>51.3</td><td>183</td></tr>
<tr><td>Stand alone</td><td>05.6</td><td>03</td></tr>
<tr><td>Proposed semi-automatic approach</td><td>42.1</td><td>38</td></tr>
<tr><td>Proposed approach + Forensic Examiner</td><td>52.9</td><td>49</td></tr>
</table>

Table 2
Number of minutiae in latent fingerprints and simultaneous latent impressions.

	Number of Minutiae	
	Each Latent Fingerprint Image	Simultaneous Latent Impression (more than one finger)
Minimum	0	14
Maximum	21	85
Average	12	46

that forensic examiners can subsequently use to manually compare and find the best match. Finally, because the approach is semi automatic, the overall time is significantly reduced.

We believe that the results of this preliminary work would motivate further research in this area. There are still several key challenges to be addressed. There is a need for a large simultaneous latent impressions database obtained from different surfaces with varying quality and quantity. Such a database will facilitate a meaningful scientific statistical study on the usefulness and applicability of simultaneous latent fingerprint impressions. The algorithm can be improved to fully automate the process for establishing simultaneity and reliably extracting level-2 and level-3 features from the latent impressions. This will further reduce the speed and perform identification even when none of the friction ridge impressions stand alone.

6 Acknowledgement

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Appendix - A: SVM [18] is a pattern classifier that constructs non-linear hyperplanes in a multidimensional space. In this research, we use dual ν-SVM (2ν-SVM) [19]. A brief overview of 2ν-SVM is presented here.

Let $\{\mathbf{x}_i, y_i\}$ be a set of N data vectors with $\mathbf{x}_i \in \Re^d$, $y_i \in (+1, -1)$, and $i = 1, ..., N$. $\mathbf{x}_i$ is the i^{th} data vector that belongs to a binary class y_i. According to Chew *et al.* [19], the objective of training 2ν-SVM is to find the hyperplane that separates the two classes with the widest margins, i.e., $\mathbf{w}\varphi(\mathbf{x}) + b = 0$ to minimize,

$$\begin{cases} \frac{1}{2}\|\mathbf{w}\|^2 - \sum_i C_i(\nu\rho - \psi_i) \\ \text{subject to } y_i\,(\mathbf{w}\,\varphi(\mathbf{x}_i) + b) \geq (\rho - \psi_i), \quad \rho,\ \psi_i \geq 0 \end{cases} \tag{.1}$$

where $\varphi(\mathbf{x})$ is the mapping function used to map the data space to the feature space and provide generalization for the decision function that may not be a linear function of the training data. ρ is the position of the margin, ν is the error parameter, $C_i(\nu\rho - \psi_i)$ is the cost of errors, w is the normal vector, b is the bias, and ψ_i is the slack variable for classification errors. The error parameter ν can be calculated using Equation (.2).

$$\nu = \frac{2\nu_+\nu_-}{\nu_+ + \nu_-}, \quad 0 < \nu_+ < 1, \quad and \quad 0 < \nu_- < 1 \tag{.2}$$

where ν_+ and ν_- are the error parameters for the positive and negative classes, respectively. Error penalty C_i is calculated as,

$$C_i = \begin{cases} C_+, & if \quad y_i = +1 \\ C_-, & if \quad y_i = -1 \end{cases} \tag{.3}$$

where,

$$C_+ = \left[n_+\left(1 + \frac{\nu_+}{\nu_-}\right)\right]^{-1}, \quad C_- = \left[n_-\left(1 + \frac{\nu_-}{\nu_+}\right)\right]^{-1} \tag{.4}$$

Here, n_+ and n_- are the number of positive and negative training samples, respectively. Finally, 2ν-SVM training [19] can be formulated as,

$$max_{(\alpha_i)}\left\{-\frac{1}{2}\sum_{i,j}\alpha_i\,\alpha_j\,y_i\,y_j\,K(x_i, x_j)\right\} \tag{.5}$$

where,

$$0 \leq \alpha_i \leq C_i, \quad \sum_i \alpha_i y_i = 0, \quad and \quad \sum_i \alpha_i \geq \nu \tag{.6}$$

$i, j \in 1, ..., N$, α_i, α_j are the Lagrange multipliers and $K(\mathbf{x}_i, \mathbf{x}_j)$ is the kernel function. One example of kernel function is the Radial Basis Function (RBF) kernel as shown in Equation (.7).

$$K(\mathbf{x}_i, \mathbf{x}_j) = exp(-\gamma||\mathbf{x}_i - \mathbf{x}_j||^2), \qquad \gamma > 0 \tag{.7}$$